PIV-plus-Feedforward Control using QUARC

Magnetic Levitation (MAGLEV)

Student Handout

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1. Objectives

The MAGLEV plant, as illustrated in Figure 1, is an electromagnetic suspension system acting on a solid one-inch steel ball. It mainly consists of an electromagnet, located at the upper part of the apparatus, capable of lifting from its pedestal and sustaining in free space the steel ball. Two system variables are directly measured on the MAGLEV rig and available for feedback. They are namely: the coil current and the ball distance from the electromagnet face. A more detailed description is provided in Reference [1].

During the course of this experiment, you will become familiar with the design and pole placement tuning of both PI current controller and PIV-plus-feed-forward ball position controller. The challenge of the present laboratory is to levitate a one-inch solid steel ball in air from the pedestal using an electromagnet. The control system should maintain the ball stabilized in mid-air and track the ball position to a desired trajectory.

Figure 1 The Magnetic Levitation Experiment

At the end of the session, you should know the following:
- How to mathematically model the MAGLEV plant from first principles in order to obtain the two open-loop transfer functions characterizing the system, in the Laplace domain.
- How to linearize the obtained non-linear equation of motion about the quiescent point of operation.
- How to design, through pole placement, a Proportional-plus-Integral (PI) controller for the MAGLEV electromagnet current in order for it to meet the required design specifications.

- How to design, through pole placement, a Proportional-plus-Integral-plus-Velocity (PIV) controller with feed-forward action for the MAGLEV levitated ball position in order for it to meet the required design specifications.
- How to implement your two controllers in real-time and evaluate their actual performances.
- How to numerically determine the system's actual closed-loop poles, by considering the coil current control system's dynamics.

2. Prerequisites

To successfully carry out this laboratory, the prerequisites are:
i) To be familiar with your MAGLEV plant main components (e.g. mechanical design, actuator, sensors), your power amplifier (e.g. VoltPAQ), and your data acquisition card (e.g. Q2-USB), as described in References [1], [2], and [3].
ii) To be familiar in using QUARC to control and monitor the plant in real-time and in designing their controller through Simulink, as detailed in Reference [4].
iii) To be familiar with the complete wiring of your MAGLEV specialty plant, as per dictated in Reference [1].

3. References

[1] MAGLEV User Manual.
[2] Data Acquisition Board User Manual.
[3] Amplifier User Manual.
[4] QUARC User Manual (type `doc quarc` in the MATLAB Command Prompt).

4. Experimental Setup

4.1. Main Components

To setup this experiment, the following hardware and software are required:

- **Power Module:**   Quanser VoltPAQ, or equivalent.
- **Data Acquisition Board:**   Quanser Q2-USB, Q8-USB, QPID, or equivalent.
- **MAGLEV Plant:**   Quanser MAGLEV, as represented in Figure 1, above.
- **Real-Time Control Software:**   QUARC or LabVIEW or equivalent.

For a complete and detailed description of the main components comprising this setup, please refer to the manuals corresponding to your configuration.

4.2. Wiring

To wire up the system, please follow the default wiring procedure for your MAGLEV as fully described in Reference [1]. When you are confident with your connections, you can power up the amplifier. The MAGLEV inside chamber should then light up.

5. Controller Design Specifications

In the present laboratory (i.e. the pre-lab and in-lab sessions), you will design and implement two control strategies in order to successfully regulate and track the ball position in mid-air.

5.1. Coil Current Controller Specifications

The first closed-loop system is to control the electromagnet coil current via the commanded coil voltage. It is based on a Proportional-plus-Integral (PI) scheme.

In response to a 0-to-1-A square wave coil current setpoint, tune the PI current controller in order to satisfy the following design performance requirements:
1. The current response should be over-damped (i.e. no overshoot).
2. Have no steady-state error.
3. Have a maximum rise time, t_{r_c}, less than 0.35 seconds, i.e.:

$$t_{r\_c} \leq 0.35 \ [\, s \,]$$

5.2. Ball Position Controller Specifications

The second and last control strategy is to regulate and track in mid-air the ball position. The closed-loop scheme employed consists of a Proportional-plus-Integral-plus-Velocity (PIV) controller with a feed-forward component.

The first specification is to design the ball position controller for the following operating position (a.k.a. equilibrium position), x_{b0}:

$$x_{b0} = 6 \ [\, mm \,]$$

In response to a desired ±1-mm square wave position setpoint from the ball equilibrium position in mid-air, the ball position behavior should satisfy the following design performance requirements:

1. Have a Percent Overshoot, PO, less than 15%, i.e.:

   $$PO \leq 15 \left[\text{"\%"} \right]$$

2. Have no static steady-state error.

3. Have a maximum settling time, t_{s_b}, less than 1.0 second, i.e.:

   $$t_{s\_b} \leq 1.0 \left[s \right]$$

4. Minimize the control effort produced, which is proportional to the coil input voltage V_c. The power amplifier should not go into saturation in any case.

6. Pre-Lab Assignments

6.1. MagLev System Representation and Notations

A schematic of the Magnetic Levitation (MAGLEV) plant is represented in Figure 2, below. The MAGLEV system's nomenclature is provided in Appendix A. As illustrated in Figure 2, the positive direction of vertical displacement is downwards, with the origin of the global Cartesian frame of coordinates on the electromagnet core flat face, as represented in Figure 2. Although the ball does have six Degrees Of Freedom (DOF) in free space, only the vertical (i.e. x) axis is controlled. It can also be seen that the MAGLEV consists of two main systems: an electrical and an electro-mechanical.
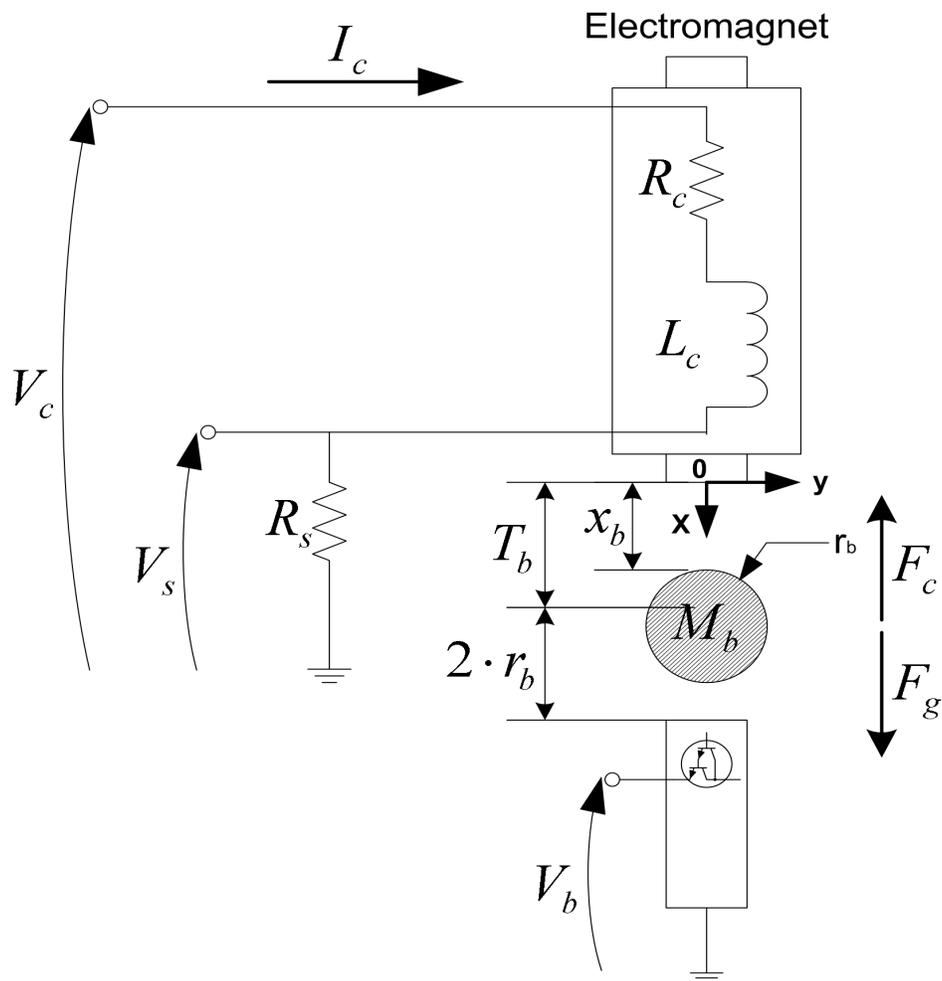

Figure 2 Schematic of the MAGLEV Plant

6.2. Assignment #1: Electrical System Modeling

Assignment #1 derives the mathematical model of your MAGLEV electrical system. The resulting model will provide you with the open-loop transfer function, $G_c(s)$, which in turn will be used to design an appropriate controller.

Equation [1] defines the Laplace open-loop transfer function, $G_c(s)$, of the MAGLEV coil voltage to the coil current. $G_c(s)$ is defined as shown below:

$$G_c(s) = \frac{I_c(s)}{V_c(s)} \qquad\qquad [1]$$

Answer the following questions:

1. As represented in Figure 2, above, the MAGLEV coil has an inductance and a resistance. Additionally, the actual system is equipped with a current sense resistor in series with the coil and whose voltage, Vs, can be measured using the data acquisition board Analog-To-Digital converter. The coil current can then be computed using the following relationship:

$$I_c(t) = \frac{V_s(t)}{R_s} \qquad\qquad [2]$$

Using the MAGLEV equivalent electrical circuit depicted in Figure 2, derive the differential equation governing the current, I_c, flowing through the actual electromagnet coil as a result of the applied voltage V_c.

2. From the previously-obtained differential equation, determine the electrical system transfer function, as defined in Equation [1]. Express the open-loop transfer function DC gain, Kc_{dc}, and time constant, τ_c, as functions of the basic electrical components. Is the system stable? What are its order and its type?

6.3. Assignment #2 - Electro-Mechanical System Modeling: Non-Linear Equation Of Motion (EOM)

Answer the following questions:

1. Using the notations and conventions described in Figure 2, above, derive the Equation Of Motion (EOM) of the MAGLEV electro-mechanical system. Is the MAGLEV electro-mechanical system's EOM linear?

Hint #1:
The attractive force, F_c, generated by the electromagnet and acting on the steel ball is assumed to be expressed as:

$$F_c = \frac{1}{2} \frac{K_m I_c^{\,2}}{x_b^{\,2}} \qquad \text{for} \quad 0 < x_b \qquad\qquad [3]$$

Equation [3] shows that the pull of the electromagnet is proportional to the square of the current and inversely proportional to the air gap (a.k.a. ball position) squared.

Hint #2:
The Newton's second law of motion can be applied to the steel ball.

Hint #3:
Express the resulting EOM under the following format:

$$\frac{\partial^2}{\partial t^2} x_b = \mathrm{f}(x_b, I_c) \qquad\qquad [4]$$

where f denotes a function.

2. The nominal coil current I_{c0} for the electromagnet-ball pair can be determined at the system's static equilibrium. By definition, static equilibrium at a nominal operating point (x_{b0}, I_{c0}) is characterized by the ball being suspended in air at a constant position x_{b0} due to a constant electromagnetic force generated by I_{c0}. Express the static equilibrium current I_{c0} as a function of the system's desired equilibrium position x_{b0} and its electromagnet force constant K_m. Using the system's specifications given in Reference[1] and the desired design requirements, evaluate I_{c0}. For reference, also express the electromagnet force constant K_m as a function of the system's desired equilibrium point (x_{b0}, I_{c0}).

6.4. Assignment #3 - Electro-Mechanical System Modeling: EOM Linearization and Transfer Function

In order to design and implement a linear position controller for our system, the Laplace open-loop transfer function can be derived. However by definition, such a transfer function can only represent the system's dynamics from a <u>linear</u> differential equation. Therefore, the EOM found in Assignment #2 should be linearized around a quiescent point of operation.

In the case of the levitated ball, the operating range corresponds to small departure positions, x_{b1}, small departure currents, I_{c1}, from the desired equilibrium point (x_{b0}, I_{c0}). Therefore, x_b and I_c can be expressed as the sum of two quantities, as shown below:

$$x_b = x_{b0} + x_{b1} \qquad \text{and} \qquad I_c = I_{c0} + I_{c1} \qquad\qquad [5]$$

Answer the following questions:

1. Linearize the ball's EOM found in Assignment #2 about the quiescent operating point (x_{b0}, I_{c0}).
   Hint:
   For a function, f, of two variables, x_b and I_c, a first order approximation for small variations at a point $(x_b, I_c) = (x_{b0}, I_{c0})$ is given by the following Taylor's series approximation:

$$f(x_b, I_c) = f(x_{b0}, I_{c0}) + \left(\frac{\partial}{\partial x_b} f(x_{b0}, I_{c0}) \right) (x_b - x_{b0}) + \left(\frac{\partial}{\partial Ic} f(x_{b0}, I_{c0}) \right) (I_c - I_{c0}) \quad [6]$$

2. Determine from the previously obtained linear equation of motion, the system's open-loop transfer function in the Laplace domain, as defined by the following relationship:

$$G_{bl}(s) = \frac{x_{bl}(s)}{I_{cl}(s)} \quad [7]$$

   Express the open-loop transfer function DC gain, Kb_{dc}, and natural frequency, ω_b, as functions of x_{b0} and I_{c0}. Is the system stable? What are its order and its type?

As a remark, it is obvious that linearized models, such as the MAGLEV current-to-position transfer function, are only approximate models. Therefore, they should be treated as such and used with appropriate caution, that is to say within the valid operating range and/or conditions. However for the scope of this lab, Equation [7] is assumed valid over the steel ball entire range of motion, T_b.

6.5. Assignment #4 – Coil Current Controller Design: Pole Placement

Prior to control the steel ball position, the current flowing through the electromagnet needs to be controlled. The electromagnet current control loop consists of a Proportional-plus-Integral (PI) closed-loop scheme, as illustrated in Figure 3, below.
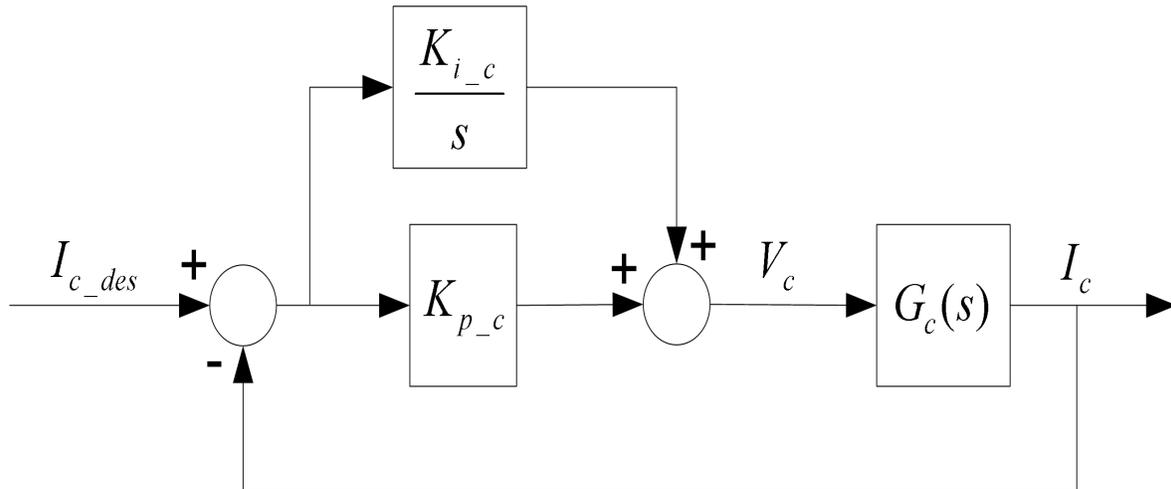
Figure 3 PI Current Control Loop

Answer the following questions:

1. Using the coil voltage-to-current transfer function, $G_c(s)$, determined in Assignment #1, derive the coil current closed-loop transfer function defined by Equation [8] below and characterized by Figure 3:

$$T_c(s) = \frac{I_c(s)}{I_{c\_des}(s)} \qquad\qquad [8]$$

Hint #1:
Block diagram reduction can be carried out.
Hint #2:
$T_c(s)$ should be a function of the current PI controller gains: K_{p_c} and K_{i_c}.

2. What is the normalized characteristic equation of the electrical system?

3. We wish to place the closed-loop poles of the current loop at the two following locations: pc1 and pc2. What would then be the system's desired characteristic equation, as a function of pc1 and pc2?

4. Determine K_{p_c} and K_{i_c} as functions of p_{c1} and p_{c2}, so that the desired closed-loop pole locations for the current loop are achieved.

6.6. Assignment #5 – Ball Position Controller Design: Pole Placement

The steel ball position is controlled by means of a Proportional-plus-Integral-plus-Velocity (PIV) closed-loop scheme with the addition of a feed-forward action, as illustrated in Figure 4, below.

As depicted in Figure 4, the current feed-forward action is characterized by:

$$I_{c\_ff} = K_{ff\_b} x_{b\_des}$$ [9]

and:

$$I_c = I_{c1} + I_{c\_ff}$$ [10]

As it can be seen in Figure 4, the feed-forward action is necessary since the PIV control system is designed to compensate for small variations (a.k.a. disturbances) from the linearized operating point (x_{b0}, I_{c0}). In other words, while the feed-forward action compensates for the ball gravitational bias, the PIV controller compensates for dynamic disturbances.
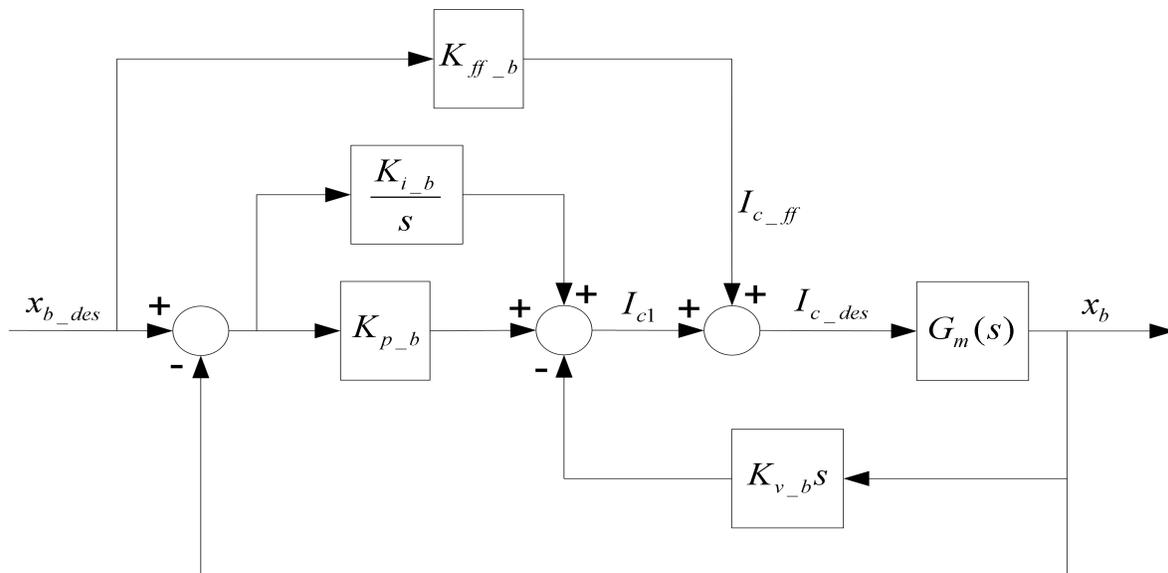

Figure 4 Ball Position PIV-plus-Feedforward Control Loop

The open-loop transfer function $G_m(s)$ takes into account the dynamics of the electromagnet

current loop, as characterized in Assignment #4, and is defined as shown below

$$G_m(s) = \frac{x_b(s)}{I_{c\_des}(s)} \qquad \text{and} \qquad G_m(s) = T_c(s)\, G_b(s) \qquad [11]$$

where:

$$G_b(s) = \frac{x_b(s)}{I_c(s)} \qquad [12]$$

However for sake of simplicity, we neglect in the following the dynamics of the electromagnet current loop. In this analysis of the ball position PIV-plus-feedforward control loop, as presented hereafter, it is therefore assumed that:

$$I_c(t) = I_{c\_des}(t) \qquad \text{i.e.} \qquad T_c(s) = 1 \qquad [13]$$

Moreover due to the presence of the feed-forward loop, we have:

$$G_b(s) = G_{b1}(s) \qquad [14]$$

where $G_{b1}(s)$ is as defined is Assignment #3.

Answer the following questions:

1. Analyze the ball position closed-loop system at the static equilibrium point (x_{b0}, I_{c0}) and determine the current feed-forward gain, K_{ff_b}.

2. Using the MAGLEV current-to-position transfer function $G_b(s)$ determined in Assignment #3, derive the ball position closed-loop transfer function defined by Equation [15] below and illustrated by Figure 4:

$$T_b(s) = \frac{x_b(s)}{x_{b\_des}(s)} \qquad [15]$$

Hint #1:
Block diagram reduction can be carried out.
Hint #2:
$T_b(s)$ should be a function of the feed-forward gain K_{ff_b} as well as of the PIV position controller gains: K_{p_b}, K_{i_b}, and K_{v_b}.

3. What is the normalized characteristic equation of the obtained electro-mechanical system?

4. We wish to place the closed-loop poles of the position loop at the three following locations: p_{b1}, p_{b2}, and p_{b3}. What would then be the system's desired characteristic equation, as a function of p_{b1}, p_{b2}, and p_{b3}?

5. Determine K_{p_b}, K_{i_b}, and K_{v_b} as functions of p_{b1}, p_{b2}, and p_{b3}, so that the desired closed-loop pole locations for the position loop are achieved.

7. In-Lab Procedure

7.1. Experimental Setup And Wiring

Even if you do not configure the experimental setup entirely yourself, you should be at least completely familiar with it and understand it. If in doubt, refer to References [1], [2], [3], and/or [4].

The first task upon entering the lab is to ensure that the complete system is wired as fully described in Reference [1]. You should be familiar with the complete wiring and connections of your MAGLEV system. If you are still unsure of the wiring, please ask for assistance from the Teaching Assistant assigned to the lab. When you are confident with your connections, you can power up the amplifier. The MAGLEV inside chamber should light up. You are now ready to begin the lab.

7.2. Real-Time Implementation: Tuning Of The PI Current Control Loop

7.2.1. Objectives

- To tune through pole placement the PI controller for the actual electromagnet current.
- To implement in real-time using QUARC, the PI control loop for the actual MAGLEV coil current.
- To run the obtained PI current controller and compare the actual response against the controller design specifications.

7.2.2. Experimental Procedure

Please follow the steps described below:

Step1. If you have not done so yet, you can start-up MATLAB now. Open the Simulink model named *q_pi_maglev.mdl.* You should obtain a diagram similar to the one shown in Figure 5, below. The model implements a Proportional-plus-Integral (PI) closed-loop, whose gains need to be calculated and initialized. The actual PI control scheme is similar to the one described in Assignment #4, as illustrated in Figure 6, below. To run your actual electromagnet current, it directly interfaces with your MAGLEV hardware, as shown in Figure 7, below. To familiarize yourself with the diagram, it is suggested that you open the model subsystems to get a better idea of

their composing blocks as well as take note of the I/O connections. You should also check that the signal generator block properties are properly set to output a square wave signal, of amplitude 1 and of frequency 0.1 Hz. The total current setpoint should result to be a square wave between 0 and 1 A.
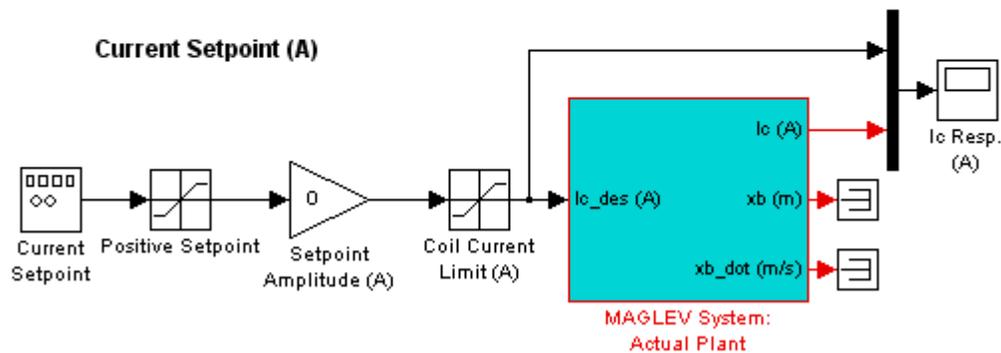
Figure 5 Real-Time Implementation of the PI Coil Current Control Loop And Setpoint Generation
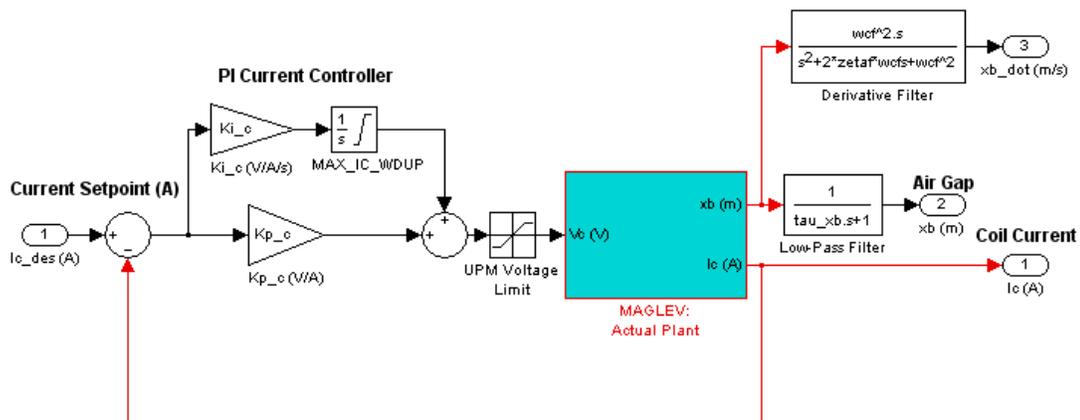
Figure 6 Real-Time Implementation of the PI Current Control Loop

Figure 7 Interface Subsystem to the Actual MAGLEV Plant.
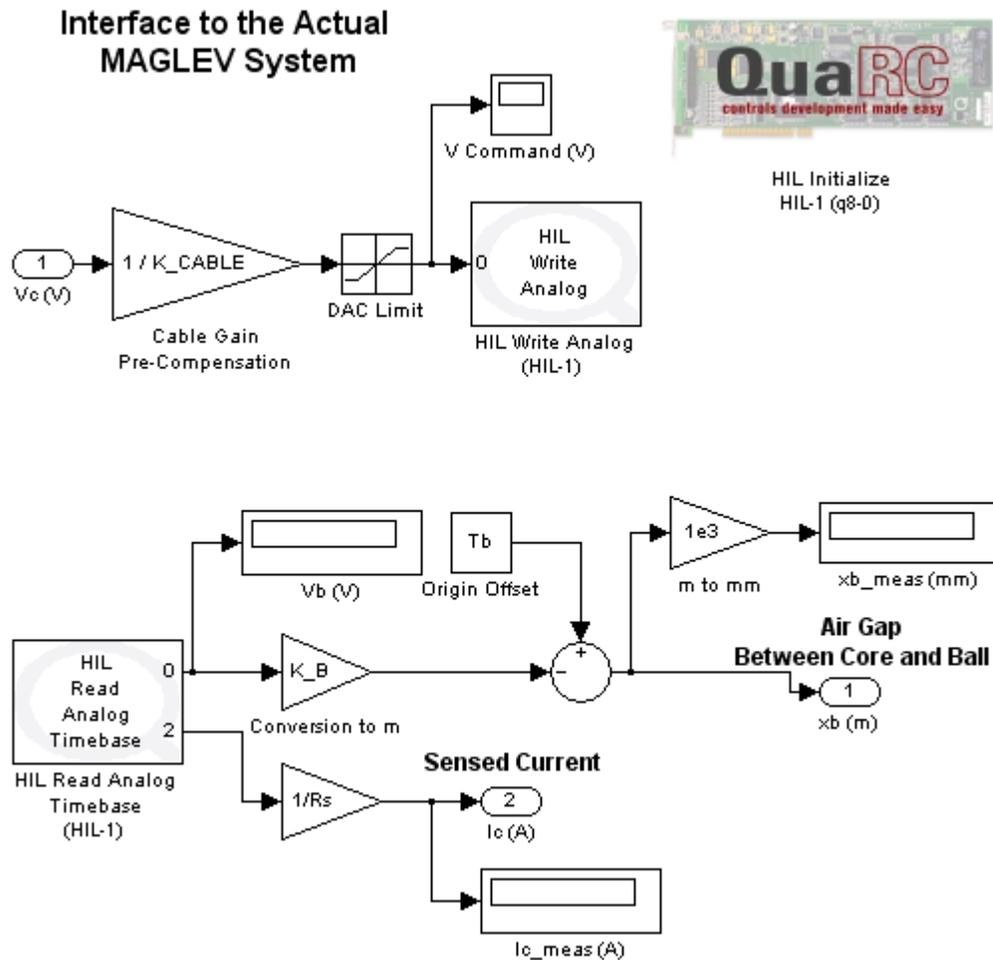
Step2. The *HIL Initialize* block should be configured depending on the type of data acquisition board that you are using (e.g. Q2-USB). By default it is set to the Q8 DAQ. However if you are using a different type of DAQ, you should configure the *HIL Initialize* block by double-clicking on it and selecting you board from the "Board type" drop down menu.

Step3. Before being able to run the actual control loop, the PI controller gains must be calculated and initialized in the MATLAB workspace, since they are to be used by the Simulink controller diagram. Start by running the MATLAB script called *setup_lab_maglev_piv.m*. However, ensure beforehand that the *CONTROLLER_TYPE* flag is set to *'MANUAL'*. This file initializes all the MAGLEV model parameters and user-defined configuration variables needed by the Simulink diagram. Given a set a closed-loop pole locations, p_{c1} and p_{c2}, you are now in a

position to calculate the PI controller gains, K_{p_c} and K_{i_c}, from the equations you derived in pre-lab Assignments #1 and 4.

Step4. According to the PI controller design specifications, the current response should be over-damped (i.e. no overshoot) and relatively fast. The zero-steady-state error requirement should be satisfied by the controller Integral action. Accordingly, closed-loop pole locations are chosen on the real-axis such as shown below:

$$p_{c1} = -235 - 70\,i\left[\frac{rads}{\sec}\right] \qquad \text{and} \qquad p_{c2} = -235 + 70\,i\left[\frac{rads}{\sec}\right] \qquad [16]$$

Step5. From your results obtained in pre-lab Assignments #1 and 4, calculate the PI controller gains, K_{p_c} and K_{i_c}, satisfying Equation [16]. Have your lab assistant check your values. With his or her approval you can now enter your calculated values for K_{p_c} and K_{i_c} in the MATLAB workspace, by following the MATLAB notations used for the controller gains as presented in Table A.2 of Appendix A. You are now ready to go ahead with running the actual PI current controller.

Step6. Build the real-time code corresponding to your diagram, by using the *QUARC | Build* option from the Simulink menu bar. Once the code is successfully built, select the *QuarRC | Start* option from the Simulink menu bar.

Step7. In order to observe the system's real-time responses from the actual system, open the following Scope: *Ic Resp. (A)*. You should now be able to monitor on-the-fly the actual coil current as it tracks the pre-defined reference input, as well as the corresponding control effort.

Step8. Assess the actual performance of the current response and compare it to the design requirements. Measure your response actual rise time. Are the design specifications satisfied? Explain. If your current response does not meet the desired design specifications of Section Controller Design Specifications on page 3, review your PI gain calculations and/or alter the closed-loop pole locations until they do. If you are still unable to achieve the required performance level, ask your T.A. for advice.

Step9. What are your final PI controller gain values? Once your results are in agreement with the closed-loop requirements, your current response should look similar to the one displayed in Figure 8, below.

Step10. Include in your lab report your final values for K_{p_c} and K_{i_c} as well as the resulting response plot of I_c versus I_{c_des}.

Step11. You can now proceed to the next section, which deals with the implementation in real-time of your PIV-plus-Feedforward position controller on the actual MAGLEV-plus-Ball system.
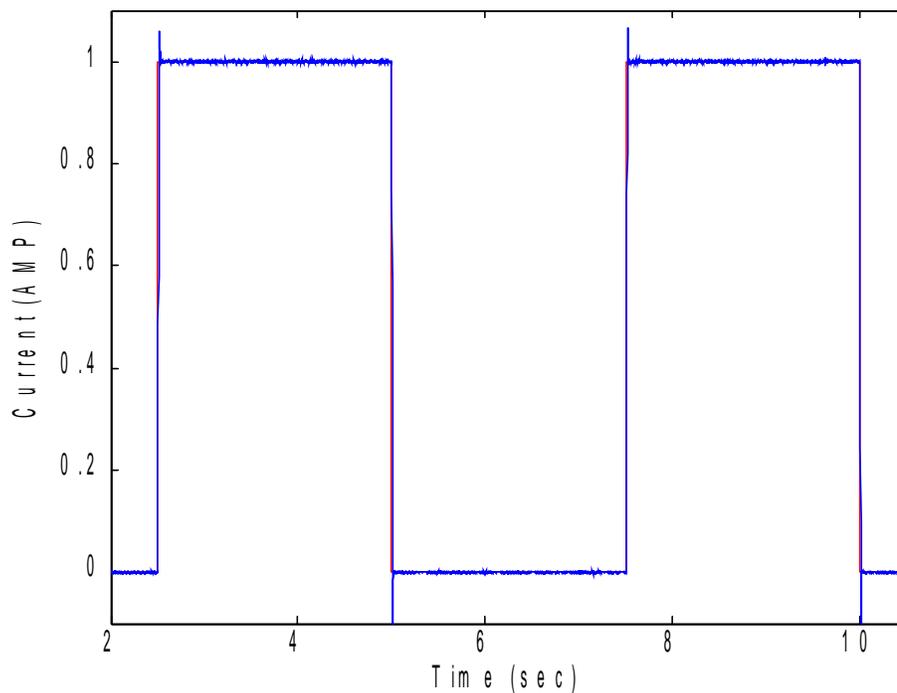
Figure 8 Actual Ic Response: PI Current Control Loop

7.3. Real-Time Implementation: Tuning Of The Feed-forward-Plus-PIV Position Control Loop

7.3.1. Objectives

- To tune through pole placement the Feed-forward-plus-PIV controller for the actual MAGLEV ball position.
- To implement in real-time with QUARC the Feed-forward-plus-PIV control loop for the actual ball position.
- To run the obtained Feed-forward-plus-PIV position controller and compare the actual response against the controller design specifications.
- To investigate the effect of the nested PI current control loop on the system's closed-loop poles.

7.3.2. Experimental Procedure

Please follow the steps described below:

Step 1. Open the Simulink model file named *q_piv_maglev.mdl.* You should obtain a diagram similar to the one shown in Figure 9, below. The model implements a Feed-forward-plus-Proportional-plus-Integral-plus-Velocity (PIV) closed-loop, as described in Assignment # 5. The four ball position controller gains need to be calculated and initialized in MATLAB, as they are used by the Simulink diagram.
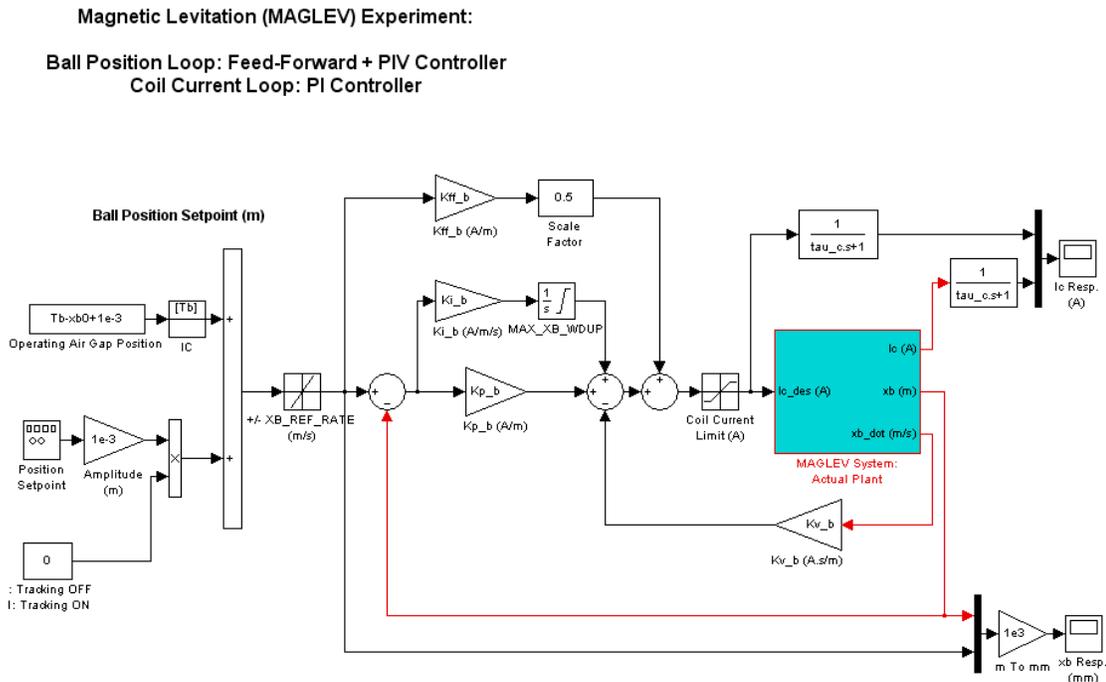

Figure 9 Diagram used for the Real-Time Implementation of the PIV Closed-Loop.

As mentioned in the pre-lab assignments, the ball position loop is based on the coil PI current controller, as developed and tuned in the previous section. The actual PI control scheme is depicted in Figure 6, above. Similarly, the position controller diagram also directly interfaces with your MAGLEV hardware, as shown in Figure 7, above. To familiarize yourself with the diagram, it is suggested that you open the model subsystems to get a better idea of their composing blocks as well as take note of the I/O connections. You should also check that the signal generator block properties are properly set to output a square wave signal, of amplitude 1, and of frequency 0.25 Hz. It should be noted that a simple low-pass filter of cut-off frequency 80 Hz (set by *'tau_xb'*) is added to the ball position sensor output signal in

order to attenuate its high-frequency noise content. Moreover, the ball vertical velocity is estimated by differentiating the ball position analog signal. Therefore, to get around potential noise problems, a second-order filter of cut-off frequency 100 Hz (set by *'wcf'*) and damping ratio 0.9 (set by *'zetaf'*) is also introduced after the differentiated velocity signal. Although introducing a short delay in the signals, low-pass filtering allows for higher controller gains in the closed-loop system, and therefore for higher performance.

Step 2. Before being able to run the actual control loop, the feed-forward and PIV controller gains must be calculated and initialized in the MATLAB workspace, since they are to be used by the Simulink controller diagram. Keep the PI current controller gain values as previously implemented. As it has been seen in pre-lab assignment #5, the quiescent feed-forward term is added to I_{c1} to compensate for the ball weight and to bring it to its operating position. Evaluate the feed-forward gain, as determined in Assignment #5, so that the design requirements are satisfied.

Step 3. The controller integral action is included to eliminate any static steady-state error. However in the case of the MAGLEV, it also compensates for the thermal drift in the coil force characteristics due to heating by electrical dissipation. Furthermore, it can be shown that the velocity (or derivative) action of the controller is crucial for the stability of the ball equilibrium position. In other words, damping appears to be critical in stabilizing the system. This "stability" consideration can justify the design requirement of having an over-damped position response. Accordingly, closed-loop pole locations are chosen to lie on the real-axis, such as shown below:

$$p_{b1} = -2.5 \left[\frac{rad}{s} \right] \quad \text{and} \quad p_{b2} = -44.0 \left[\frac{rad}{s} \right] \quad \text{and} \quad p_{b3} = -51.6 \left[\frac{rad}{s} \right] \quad [17]$$

Step 4. Given the set of desired closed-loop pole locations, p_{b1}, p_{b2}, and p_{b3}, presented by Equation [17], calculate the PIV-plus-feedforward position controller gains K_{ff_b}, K_{p_b}, K_{v_b}, and K_{i_b} from the equations you derived in pre-lab Assignment #5. Have your lab assistant check your values. With his or her approval you can now enter your calculated values for K_{ff_b}, K_{p_b}, K_{v_b}, and K_{i_b} in the MATLAB workspace, by following the MATLAB notations used for the controller gains as presented in Table A.2 of Appendix A. You are now ready to go ahead with running the actual PIV-plus-feed-forward current controller.

Step 5. If absent, place the steel ball on the pedestal inside the MAGLEV chamber.

Step 6. Build the real-time code corresponding to your diagram, by using the *QUARC | Build* option from the Simulink menu bar. Once the code is successfully built, select the *QUARC | Start* option from the Simulink menu bar.

Step 7. This should levitate the ball halfway between the post and the electromagnet core face, with an air-gap of x_{b0}. If the ball is swaying left and right, you can gently put your hand in and dampen the motion. Once the ball is stabilized at the operating point, change the value of the Simulink constant block named *0: Tracking OFF; 1:*

Tracking ON from 0 to 1. The ball should start tracking the desired ±1-mm square wave setpoint around the operating position.

Step 8. In order to observe the system's real-time responses from the actual system, open the three following scopes: *xb Resp. (mm)*, *Ic Resp. (A)*, and *V Command (V)* scope located, for example, in the following subsystem path: *MAGLEV System: Actual Plant/MAGLEV: Actual Plant*. You should now be able to monitor on-line, as the ball moves, the actual ball position, coil current, and command voltage (proportional to the control effort) sent to the power amplifier, as the ball tracks the pre-defined reference input.

Step 9. Assess the actual performance of the ball position response and compare it to the design requirements. Measure the response actual settling time. Are the design specifications satisfied? Explain. If your position response does not meet the desired design specifications of Section Controller Design Specifications on page 3, review your feed-forward and PIV gain calculations and/or alter the closed-loop pole locations until they do. If you are still unable to achieve the required performance level, ask your T.A. for advice.

Step 10. What are your final feed-forward and PIV controller gain values? Once your results are in agreement with the closed-loop requirements, your position response should look similar to the one displayed in Figure 10, below.

Note:

Fluctuations in the position signal can be due to actual fluctuations of the ball vertical position but also to swaying (due to the hemispheric surface of the ball).
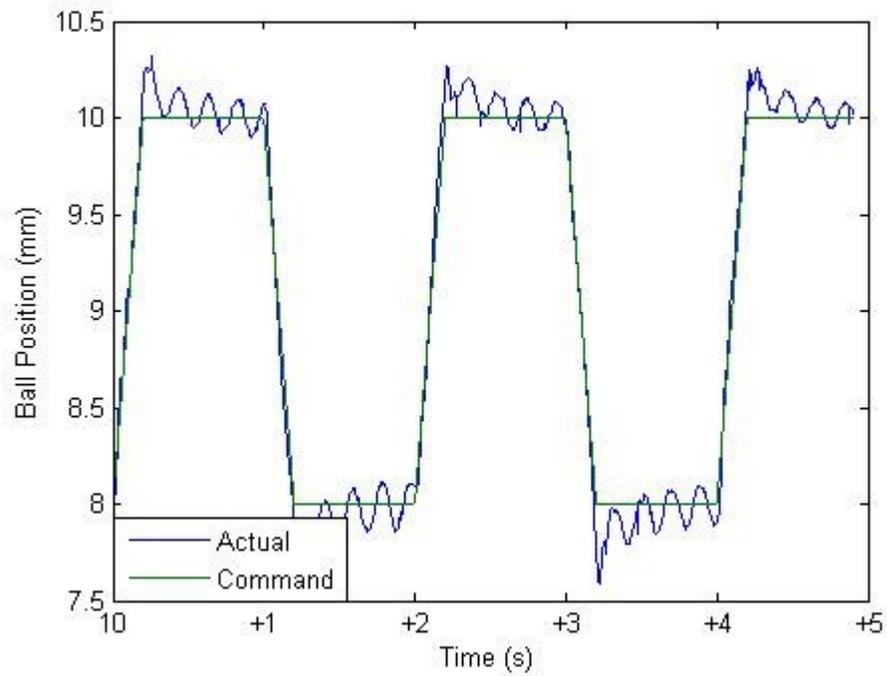
Figure 10 Actual Ball Position Tracking Response

Step 11. From the same run, the corresponding coil current is displayed in Figure 11.
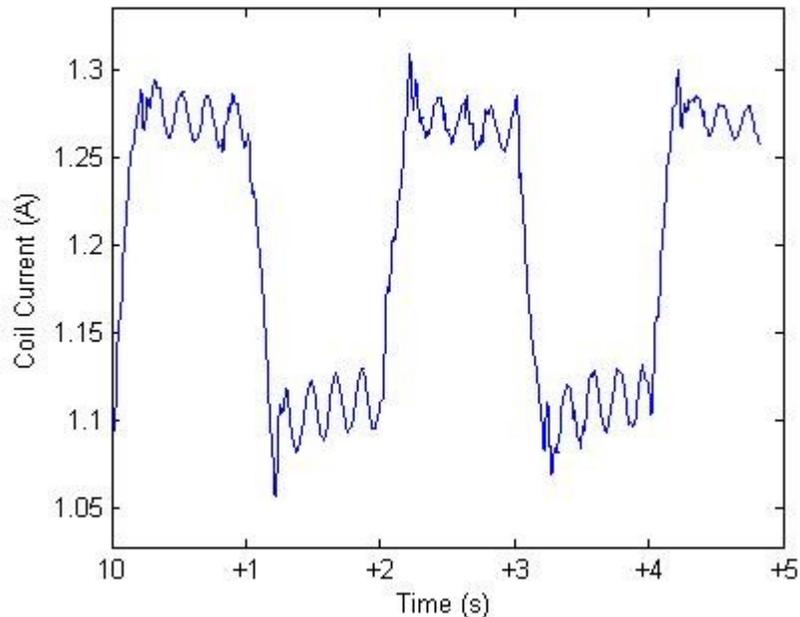
Figure 11 Actual Coil Current Response

Step 12. Include in your lab report your final values for K_{ff_b}, K_{p_b}, K_{i_b}, and K_{v_b}, as well as the resulting response plot of x_b versus x_{b_des}, and the corresponding plots of I_c and $V_{command}$. Describe the ball position variations (a.k.a. jitter) about the desired ball position as well as its instantaneous and static steady-state errors. Describe the corresponding current and voltage behaviours. Is there any saturation in the system? Ensure to properly document all your results and observations before moving on the the next section.

Step 13. You can now proceed to the next section.

7.4. Resulting System's Actual Closed-Loop Poles

7.4.1. Objectives

- To numerically determine the system's actual closed-loop poles, by considering the coil current control system's dynamics previously neglected.
- To carry out block diagram reduction using some of the MATLAB's Control System Toolbox functions.

7.4.2. Experimental Procedure

Please follow the steps described below:

Step 1. We now consider the dynamics of the electromagnet current loop, contrary to our initial simplifying assumption of Equation [13]. Using your expression for $T_c(s)$ derived in Assignment #4, evaluate with MATLAB $G_m(s)$ as defined in Equation [11].

Step 2. Let us define $T_m(s)$ as the MAGLEV rig overall closed-loop transfer function. It consists of the feedforward-plus-PIV position control scheme applied to $G_m(s)$. Using some of the MATLAB's Control System Toolbox functions, carry out block diagram reduction in order to evaluate $T_m(s)$, which is the system's actual closed-loop transfer function .

Hint:
The MATLAB Control System Toolbox functions of interest include: *'tf'*, *'series'*, *'parallel'*, *'feedback'*.

Step 3. Use some of the MATLAB's Control System Toolbox functions to calculate the system's actual closed-loop zeros and closed-loop poles.

Hint:
The MATLAB Control System Toolbox functions of interest include: *'zero'*, *'pole'*.

Step 4. Compare the actual closed-loop poles with their design value counterparts. Explain.

Step 5. You can now move on to writing your lab report. Ensure to properly document all your results and observations before leaving the laboratory session.

Appendix A. Nomenclature

Table 1, below, provides a complete listing of the symbols and notations used in the MAGLEV-and-Ball mathematical modeling, as presented in this laboratory. The numerical values of the system parameters can be found in Reference [1].

Symbol	Description	Units	MATLAB Notations
L_c	Coil Inductance	mH	Lc
R_c	Coil Resistance	Ω	Rc
N_c	Number Of Turns in the Coil Wire		Nc
r_c	Coil Core Radius	m	rc
R_s	Current Sense Resistance	Ω	Rs
r_b	Steel Ball Radius	m	rb
M_b	Steel Ball Mass	kg	Mb
T_b	Steel Ball Travel	m	Tb
g	Gravitational Constant on Earth	m/s^2	g
μ_0	Magnetic Permeability Constant	H/m	mu0
K_B	Ball Position Sensor Sensitivity	m/V	K_B
F_c	Electromagnet Force	N	Fc
F_g	Gravity Force	N	Fg
K_m	Electromagnet Force Constant	$N.m^2/A^2$	Km
I_c	Actual Coil Current	A	Ic
V_c	Actual Coil Input Voltage	V	Vc
V_s	Current Sense Voltage	V	Vs
V_b	Ball Position Sensor Voltage	V	Vb
x_b	Actual Air Gap Between Core Face and Ball Surface (a.k.a. Steel Ball Vertical Position)	m	xb
$\frac{\partial}{\partial t} x_b$	Steel Ball Vertical Velocity	m/s	xb_dot
x_{b0}	Steady-State Air Gap	m	xb0

Symbol	Description	Units	MATLAB Notations
I_{c0}	Steady-State Coil Current	A	Ic0
x_{b1}	Small Variation Around the Steady-State Air Gap	m	xb1
I_{c1}	Small Variation Around the Steady-State Coil Current	A	Ic1
I_{c_des}	Desired Coil Current	A	Ic_des
x_{b_des}	Desired Air Gap	m	xb_des
I_{c_ff}	Feed-forward Coil Current	A	Ic_ff

Table 1: Maglev System Model Nomenclature

Table 2, below, provides a complete listing of the symbols and notations used in the design of both control loops (i.e. PI and PIV-plus-Feed-forward), as presented in this laboratory.

Symbol	Description	MATLAB / Sim-ulink Notation
t_{r_c}	Coil Current Rise Time	
t_{s_b}	Ball Position Settling Time	
G_c	Open-Loop Coil Current Transfer Function	Gc
Kc_{dc}	Coil Open-Loop DC Gain	Kc_dc
τ_b	Coil Open-Loop Time Constant	tau_c
T_c	Closed-Loop Coil Current Transfer Function	Tc
G_b	Open-Loop Ball Position Transfer Function	Gb
Kb_{dc}	Ball Open-Loop DC Gain	Kb_dc
ω_b	Ball Open-Loop Natural Frequency	wb
T_b	Closed-Loop Ball Position Transfer Function	Tb
G_m	MAGLEV Rig Open-Loop Transfer Function	Gm
T_m	MAGLEV Rig Overall Closed-Loop Transfer Function	Tm
K_{p_c}	Current Proportional Gain	Kp_c
K_{i_c}	Current Integral Gain	Ki_c

Symbol	Description	MATLAB / Simulink Notation
p_{c1}	Current Dominant Closed-Loop Pole	pc1
p_{c2}	Current Second Closed-Loop Pole	pc2
K_{p_b}	Ball Position Proportional Gain	Kp_b
K_{v_b}	Ball Position Velocity Gain	Kv_b
K_{i_b}	Ball Position Integral Gain	Ki_b
K_{ff_b}	Ball Position Feed-Forward Gain	Kff_b
p_{b1}	Ball Position Dominant Closed-Loop Pole	pb1
p_{b2}	Ball Position Second Closed-Loop Pole	pb2
p_{b3}	Ball Position Third Closed-Loop Pole	pb3
t	Continuous Time	

Table 2: Control Loops Nomenclature