$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$ binormal distr. $k \in [0, n]$

Note Title 11/12/2013

$$G_X(z) = \sum_{k=0}^{\infty} P(X=k) \cdot z^k$$

$$= \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} \cdot z^k$$

$$G_X(z) = \sum_{k=0}^{\infty} (1-p)p^k z^k = \frac{1-p}{1-pz},$$

$$= (1-p) \sum_{k=0}^{\infty} (pz)^k = (1-p) \frac{1}{1-pz}$$

$$\sum_{k=0}^{\infty} (\lambda z)^k / k! = e^{\lambda z} \Leftarrow \sum_{k=0}^{\infty} \frac{\alpha^k}{k!} = e^{\alpha}$$

$$\frac{dG_X(z)}{dz} = \sum_{k=1}^{\infty} k p_k z^{k-1}$$

$$\left. \frac{dG_X(z)}{dz} \right|_{z=1} = \left. \sum_{k=1}^{\infty} k p_k z^{k-1} \right|_{z=1} = \sum_{k=1}^{\infty} k p_k = E[X]$$

$$\left.\frac{d^2G_X(z)}{dz^2}\right|_{z=1} = E[X^2] - E[X] \qquad \frac{dG_X(z)}{dz} = \sum_{k=1}^{\infty} kp_k z^{k-1} \qquad \sum_{k=0}^{\infty} p_k z^k$$

$$\hookrightarrow \frac{d\left(\sum_{k=1}^{\infty} kp_k z^{k-1}\right)}{dz} = \left.\sum_{k=2}^{\infty} k(k-1)\, kp_k\, z^{k-2}\right|_{z=1}$$

$$= \sum_{k=2}^{\infty} k(k-1)p_k = \sum_{k=2}^{\infty} (\underbrace{k^2 p_k}_{E[X^2]} - \underbrace{kp_k}_{E[X]})$$

Geometric.

$$G_X(z) = \sum_{k=0}^{\infty} (1-p)p^k z^k = \frac{1-p}{1-pz},$$

$$\frac{dG_x(z)}{dz} = \frac{d\left((1-p)\cdot(1-pz)^{-1}\right)}{dz} = (1-p)\cdot(-1)\cdot(1-pz)^{-2}\cdot(-p)$$

$$= \frac{p(1-p)}{(1-pz)^2}$$

$$E[X] = \left.\frac{dG_x(z)}{dz}\right|_{z=1} = \frac{p(1-p)}{(1-p)^2} = \frac{p}{1-p}$$

continuous r.v X $\quad f_X(x)$:

$$E[X] = \int_0^\infty f_X(x)\, x \, dx$$

$$E[e^{-sX}] = \int_0^\infty f_X(x)\, e^{-sx} \, dx$$